On the Stability of Black Holes at the LHC

M. D. Maia*
Universidade de Brasilia, Instituto de Física, Brasilia, 70910-970
&
E. M. Monte†
Universidade Federal da Paraíba, Departamento de Física, 8059-970

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Abstract

The eventual production of mini black holes by proton-proton collisions at the LHC is predicted by theories with large extra dimensions resolvable at the Tev scale of energies. It is expected that these black holes evaporate shortly after its production as a consequence of the Hawking radiation. We show that for theories based on the ADS/CFT correspondence, the produced black holes may have an unstable horizon, which grows proportionally to the square of the distance to the collision point.

The production of mini black holes at the LHC Tev energy scale is predicted by theories with large extra dimensions and where the gravitational constant is replaced by a Tev scale fundamental constant [1]. In those theories the gauge interactions are confined to four-dimensional space-times embedded in a bulk space defined by the Einstein-Hilbert principle, but gravitation as described by the four-dimensional embedded geometry, can access the extra dimensions at the TeV energy. Thus, the generated black hole will represent a local deformation of Minkowski's space-time resulting from the proton-proton collision. Typical examples are given by the Randall-Sundrum models of brane-world theory defined in the 5-dimensional anti deSitter AdS_5 bulk, using the Israel-Lanczos jump condition at the brane-world, acting as a boundary [2].

The produced black holes are expected to be short lived as consequence of Hawking's radiation. The well known theorem by Hawking of 1975, made use of semi-classical gravitation to prove that virtual particle pairs are formed in the vicinity of a black hole. While one of the particles of the pair falls inside the black hole horizon, the other escapes to infinity producing a thermal radiation of the black hole. Admitting that the radiation is complete the black hole will evaporate away, with the consequence is that the correlation between the spin states of the particles pair is lost, compromising the quantum unitarity [3]. In a recent (2005) revision of his theorem, Hawking made use of the ADS/CFT correspondence within the context of the Horava-Witten $AdS_5 \times S^5$ model of string theory. It was assumed that the black holes in question are charged (like for example, the Reissner-Nordstrom black hole), so that after the evaporation a stable submanifold would remain, in which case some of the information would be stored and could be partially retrieved [4].

In any of the above mentioned models using the same AdS_5 space, the stability of the black hole, seen as a subspace embedded in that bulk must be understood.

Consider the four-dimensional space-time AdS_4, regarded as a hypersurface with negative constant curvature embedded in the five dimensional flat space $M_5(3,2)$, with maximal isometry $SO(3,2)$ (see eg [5, 6]). Adding to each of these spaces one extra spatial dimension, we obtain in a very trivial way that the five-dimensional anti de Sitter space AdS_5 is a hypersurface with

*maia@unb.br
†Edmundo@fisica.ufpb.br

negative constant curvature embedded in the flat space $M_6(4,2)$, with maximal isometry $SO(4,2)$, which is isomorphic to the conformal group C_0 acting on the Minkowski space-time, so that for each conformal covariant Yang-Mills field in M_4, there corresponds an isometric covariant Yang-Mills field defined in the AdS_5 space.

Since all Yang-Mills fields and their duality properties are consistently defined only in four-dimensional space-times, where 3-forms are isomorphic to 1-forms, the ADS/CFT correspondences must be hold between four-dimensional subspaces of the AdS_5 bulk. Consequently, the isometric invariant Yang-Mills fields must remain confined to a 4-dimensional subspace embedded in the AdS_5 bulk, where the required duality operation is preserved. Furthermore, admitting that the embedding of the 4-dimensional subspace is local and regular, then the inverse function theorem establishes a local 1:1 correspondence between the conformal fields in M_4 and the Yang-Mills fields defined in those four-dimensional subspaces.

The AdS_5 bulk is a solution of the 5-dimensional vacuum Einstein's equations with negative cosmological constant

$$ {}^5\mathcal{R}_{AB} - \frac{1}{2}\,{}^5\mathcal{R}\mathcal{G}_{AB} - \Lambda\mathcal{G}_{AB} = 0, \quad A,B = 1...5 \tag{1} $$

and with constant curvature, characterized by the Riemann tensor

$$ {}^5\mathcal{R}_{ABCD} = \frac{\Lambda}{6}(\mathcal{G}_{AC}\mathcal{G}_{BD} - \mathcal{G}_{AD}\mathcal{G}_{BC}) \tag{2} $$

The embedding is an application $X : V_4 \to AdS_5$, with components $X^A(x_\mu)$, functions of the space-time coordinates x_μ. Together with the unit normal vector η, this defines a Gaussian reference frame on the embedded geometry $\{X^A_{,\mu}, \eta^A\}$. The Riemann tensor of the bulk written in this reference gives the integrability equations for the embedding, the well known Gauss-Codazzi equations [6], which in the case of a constant curvature 5-dimensional bulk can be written as

$$ R_{\alpha\beta\gamma\delta} = -(k_{\alpha\gamma}k_{\beta\delta} - k_{\alpha\delta}k_{\beta\gamma}) + \frac{\Lambda}{6}(g_{\alpha\gamma}g_{\beta\delta} - g_{\alpha\delta}g_{\beta\gamma}) \tag{3} $$

$$ k_{\alpha[\beta;\gamma]} = 0, \tag{4} $$

The existence of the embedding requires that the metric $g_{\mu\nu}$ and the extrinsic curvature $k_{\mu\nu}$ of V_4 satisfy these equations. Writing (1) in the same Gaussian frame we obtain the gravitational equations of the embedded V_4 [7]

$$ R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} - \Lambda g_{\alpha\beta} + Q_{\alpha\beta} = 0 \tag{5} $$

$$ k^{\gamma}_{\beta;\gamma} - h_{,\beta} = 0 \tag{6} $$

where $h = g^{\mu\nu}k_{\mu\nu}$ is the mean curvature, $K^2 = k_{\mu\nu}k^{\mu\nu}$ is the Gaussian curvature and

$$ Q_{\alpha\beta} = (k^{\rho}{}_{\alpha}k_{\rho\beta} - hk_{\alpha\beta}) - \frac{1}{2}(K^2 - h^2)\ g_{\alpha\beta}, \quad Q^{\alpha\beta}{}_{;\beta} = 0 $$

These equations describe the dynamics of the gravitational field for a space-time embedded in the AdS_5 bulk. Clearly they are more general than the vacuum Einstein's equations in general relativity, because (1) implies that the extrinsic curvature $k_{\mu\nu}$ is also a dynamical variable. When $k_{\mu\nu} = 0$ the usual vacuum Einstein's equations are recovered.

Now, we may discuss the production of black hole at the LHC with the supposition that it happens within the AdS_5 bulk. The experiment is supposed to take place in the Minkowski's space-time M_4 where the protons are defined. However, this M_4 is regarded as a subspace embedded in the AdS_5, so that equations (3) and (4) must apply. The general solution of these equation for a flat space gives $k_{\mu\nu} = \sqrt{\Lambda/6}\ \eta_{\mu\nu}$. This means that although M_4 is flat in the Riemann sense, as an embedded geometry it is also warped like a cylinder, a cone or any ruled surface, where the full

translational group of the Poincaré symmetry in principle would not apply. However, from (5), it follows that Λ is the cosmological constant, which is too small to mark a significant presence in the local gravitational field of the LHC experiment. Therefore, for any practical purposes we may assume that the experiment starts very approximately as planned, in Minkowski's space-time.

After the collision, the produced black-holes will represent a deformations of the original Minkowski's space-time, transforming into Schwarzschild or Reissner-Nordstrom subspaces embedded in the AdS_5 bulk. This black hole should remain for a very short period, before its eventual evaporation. For a spherically symmetric diagonal metric, the general solution of (4) is of the form $k_{\mu\nu} = \alpha(r)g_{\mu\nu}$, where $\alpha(r) = 1/y(r)$ and where $y(r)$ is the local center of curvature of the embedded geometry, which is a value of the extra coordinate satisfying the condition $\det(g_{\mu\nu} - y(r)k_{\mu\nu}) = 0$ [6]. Replacing this expression in (5), we obtain the Schwarzschild-anti-de Sitter solution

$$ds^2 = (1 - \frac{2m}{r} + (3\alpha(r)^2 - \Lambda)r^2)^{-1}dr^2 + r^2 d\omega^2 \quad - (1 - \frac{2m}{r} + (3\alpha(r)^2 - \Lambda)r^2)dt^2$$

Repeating the same for the Reissner-Nordstrom solution, we obtain the Reissner-Nordstrom-anti-de Sitter solution

$$ds^2 = (1 - \frac{2m}{r} + \frac{q}{r^2} + (3\alpha(r)^2 - \Lambda)r^2)^{-1}dr^2 + r^2 d\omega^2 - (1 - \frac{2m}{r} + \frac{q}{r^2} + (3\alpha(r)^2 - \Lambda)r^2)dt^2$$

In both cases we obtain a black holes whose horizon grows indefinitely with r^2, thus creating an unstable situation unless we impose the additional condition that $3\alpha(r)^2$ is constant and equal to Λ. However, this implies that the black hole is composed of umbilicus points (all of its directions look like the same principal directions), which is not the case of a black hole. Even if we neglect the local influence of Λ as before the collision, we cannot neglect $\alpha(r)$ on account of the characteristics of a black hole geometry.

We conclude that the exterior gravitational field of a Black hole is not native of an AdS_5 bulk and that the black holes produced by proton-proton collision at the LHC may be unstable. Nonetheless, it is possible that in a higher dimensional bulk $D > 5$, the behavior of the black holes is stable. This follows from the well known example given by the 6-dimensional flat bulk $M_6(4,2)$, whose metric is also invariant under $SO(4,2)$, so that it has the same group of isometries of the AdS_5. Consequently, all arguments of the ADS/CFT correspondence which depend only on the Lie group properties, can be extended without loss of generality to that flat bulk. By the same argument used in the ADS/CFT correspondence, the quantum unitarity of the Yang-Mills fields is maintained in the six-dimensional flat bulk.

References

[1] S. Dimopoulos and L. G. Landsberg, Phys. Rev. Lett. 15 October (2001).

[2] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370,(1999)

[3] S. Hawking, Nature **248**, 30 (1974), S. Hawking, Comm. Math. Phys.**43**, 199 (1975)

[4] S.Hawking, hep-th/0507171, J. Baez, www.math.ucr.edu/home/baez/week207

[5] J. Rosen, Rev. Mod. Phys. 37, 215 (1965)

[6] L. P. Eisenhart, Riemannian Geometry, Princeton, N.J. (1966)

[7] M. D. Maia et al, Class.Quant.Grav. **22**, 1623, (2005), astro-ph/0403072